Application of a Two-Sex Marriage Model to Norwegian Data

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Preface

I would like to thank my supervisor at the Department of Economics, Nico Keilman in University of Oslo for his inspirational suggestion and kind encouragement during the work on my thesis. Without his support, it is impossible to finish my thesis on time.

I also wish to thank my family and my friend Lin Ling for their help.

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1 Summary

The objective of my thesis is to compute several marriage indicators that are included in a new two-sex marriage model recently proposed by Alho and Keilman (see Alho and Keilman 2008). The model deals not only with the competition effect between marriage candidates of the same sex, but is also parameterized in terms of the overall level of nuptiality and the quality of spouses. For the two- sex marriage model, I compute maximum likelihood estimates of marriage rates, marriage intensities, gender quality parameters and a parameter which describes unbalances between potential spouses. This model is satisfactory from the actual marriage behavioral point of view. Some of the empirical results will be compared with the results obtained from a traditional one-sex marriage model by using data for annual numbers of marriages in Norway for the period 1974-2002. I compare estimates for all marriages (i.e. both first and higher order marriages) with those obtained when the data are restricted to a first marriage for both partners. Although the patterns of indicators are very similar, the precise values of them reveal different results.

2 Introduction

The traditional one-sex marriage table (one-sex marriage model) rests on a one-sex theory represented by single sex marriage rates for the single sex population. It is recognized that when there are substantial differences in marriage behavior between men and women, although widely used, the one-sex model may lead to unrealistic predictions when we ignore the contribution of the other sex. At the extreme, the model when applied to one sex, may predict a certain number of marriages even when there are zero unmarried persons of the other sex. Explicitly stated, the traditional one-sex marriage model does not take account of the interaction between the sexes which means that the behaviors of men and women are treated separately. Therefore, the one-sex marriage model can not impose consistent matching and marriage behavior across two genders. It is difficult to give a precise interpretation of key concepts and parameters in the one-sex marriage model from the perspective of single sex. A realistic marriage model should describe the behavior of the two sexes jointly.

A person's propensity to marry, operationalized by means of a marriage rate, depends on age as well as the relative number of available partners, therefore, the two-sex problem leads to special requirements for the nuptiality model. In other words, a model of the marriage market should capture the characteristics of men and women simultaneously. The two-sex problem was already discussed by several researchers; they have proposed different types of models. Although these authors have made seminal contributions to the literature on two-sex marriage models, many of the proposed marriage models are nonetheless unsatisfactory from a real marriage point of view. So far there has not been general agreement among demographers about the best way to model the marriage behavior of the two sexes simultaneously, and new models are proposed regularly.

The new two-sex marriage model proposed by Juha Alho and Nico Keilman is parameterized in terms of the overall level of nuptiality, the relative quality of the potential spouses, and the mutual relative attraction of spouses in different ages. The actual intensity of marriage depends on the available spouses. This is mediated by the harmonic mean of the population sizes of the eligible individuals. The model displays genuine competition effects between marriage candidates. Details are given in Chapter 3.

Norwegian data for the period 1974 to 2002 are applied to estimate this model. Based on the empirical analysis, the trends of those indicators are consistent all the time. The explicit values of indicators provide some important empirical results. The curves of age-specific marriage rates and gender quality parameters by age shift to higher ages between 1974 and 2002. At the same time one notices declining marriage intensities. Female intensities are higher than male intensities for the whole period. Actually, the overall marriage intensity depends on the numbers of available spouses.

To compare the results of the two sex model with corresponding parameters of the one sex model is the other objective in my thesis. The comparison will be restricted to first marriages only. In the one sex model, one key parameter is the life-time first marriage probability. This probability cannot exceed one, and it is interpreted as the probability of ever experiencing a first marriage. The detailed argument will be outlined in Chapter 4.

The thesis is organized as follows: In Chapter 3, I introduce some earlier marriage models relevant to the two-sex problem, and I outline the theoretical point of departure and the structure of the two-sex marriage model proposed by Alho and Keilman. In Chapter 4, firstly, I address a simple way to calculate the marriage intensity of the Alho-Keilman Model and describe the data. Secondly, I report the empirical results. Chapter 5 contains my conclusions.

3 The Marriage models

3.1. Some Earlier Marriage models

In this section of the thesis we shall outline some marriage formation models. We shall analyze the conditions under which they yield predictions and mention some of their qualitative properties. Many details of the models will be omitted here. The readers not familiar with particular models may consult the references cited.

Louis Henry (1968, 1972) published a number of papers to set forth his two-sex nuptiality model. He models the processes of meeting members of the other sex. He points out that marriage does not happen in some nationwide marriage market, but it takes place in some small groups named the circles. He assumes that each of the circles has the panmictic property, which means that marriage occurs in a manner that happens to be stochastic. Marriages take places in different circles with different age compositions. The aspects of both sexes are considered simultaneously in each particular circle, and thus the ages of the two genders are statistically independent of each other. His model distinguishes four stages in the process towards marriages: firstly, development of a conscious desire for marriage; secondly, joining a circle which corresponds to the tastes of the candidate; thirdly, formation of couples within these circles; and fourthly, marriage of the couples that have formed. Henry's approach mostly is based on the perception of the population being divided into several different circles. However, the circles of Henry's model are not directly observed, he treats only data aggregated to the national level, and from such data, he must infer the nature of those circles. The demographer is not at liberty to experimentally manipulate this model.

J.H.Pollard proposed a model in 1975.

$$\Phi(\mathrm{x}_1, \mathrm{x}_2) = \; k_{x_1 x_2} V_1(x_1) V_2(x_2) / \boldsymbol{T} \qquad j = 1, 2$$

In this case, we consider continuous age, and assume that marriage can occur in age 0$\leq x < \omega$. Define V_j equal to the size of unmarried population of sex j = 1, 2. A subscript 1 is used to denote males and a subscript 2 to denote females. $\boldsymbol{T}$ denotes the total population size including everyone regardless of age, sex, or marital status. Φ represents the marriage function which shows all the elements as a function of $V_1(x_1)$ and $V_2(x_2)$, where each element refers to a pair of ages. $\{k_{x_1x_2}\}$ is a parameter which links the various age-combinations in terms of the available spouses.

Schoen (1977) postulated a very similar marriage model:

$$\Phi(\mathrm{x}_1, \mathrm{x}_2) = k_{x_1x_2}\, V_1(x_1)V_2(x_2)/\boldsymbol{T}^*$$

$\boldsymbol{T}^*$ denotes the total population of marriageable age. These two models, allow for the effect of competition within the marriage market (an extra supply of single men of a certain age should decrease the marriage chances of men at other ages, and similarly for women; see below). But they ignore the problem that men in a certain age group are not equally perfect substitutes in mate selection for men of other age groups (the substitution effect; see below).

McFarland (1975) formulated an iterative adjustment model in discrete time and considered single males and females subdivided into categories x_1 and x_2 which were more general than simply age. His model proceeds from the sociological assumption that any one society has norms governing such matters as the distribution of age at marriage among members of each sex, the distribution of age of each sex among marriages involving the other sex of any particular age group. These norms are conceptualized in terms of probability distributions, rather than deterministically and taken to govern the initial search behavior of marriage candidates.

The number of possible marriages between men in the x_1-th age group and women in the x_2-th age group is assessed as a function of the number of men aged x_1 desiring marriage with women aged x_2 and the number of women aged x_2 desiring marriage with men aged x_1. After assessing the marriage market, each sex adjusts its demand for the other and the procedure is repeated. The process continues until the two sexes agree in their demand for each other. The number of marriages is determined. The form is determined by the total number of unmarried men aged x_1 and the norms regarding marriage of such men, while the latter is determined by the total number of unmarried women aged x_2 and the norms regarding marriage of such women, and thus the two vary independently rather than being constrained to have equal values. But while the number of two gender seekers for a particular type of marriage may differ, in a monogamous society the numbers actually marrying must be identical.

Applied statisticians have long used a procedure for adjusting the entries of a matrix as to change its marginal totals as desired while leaving its cross-product ratios unchanged (e.g. Deming 1943). Nowadays, the method is known as the IPF (Iterative Proportional Fitting) method. McFarland set forth the model with detailed description of the adjustments needed to bring the initial search behavior of each sex into line with the availability of candidacies of the opposite sex; this is similar to adjusting the cell frequencies of a cross-table to bring them into line with given marriage totals using the IPF algorithm. This model also employs an iterative procedure, making it difficult to assess whether the model fulfills the competition and substitution requirements. The adjustments proceed through the matrix one row at a time, multiplying each element in a row by the ratio of the desired to the current total of that row, after which the row's new entries will sum to the desired row total. When this has been done to each row in turn, the matrix is adjusted column wise. After this has been done, the columns will each sum to the desired total, but the process of adjusting the columns will have disturbed the row totals, and they will no longer agree with the desired row totals. This process is repeated, until the point is reached where rows and columns simultaneously sum to the desired totals.

McFarland postulated seven axioms which a realistic marriage model should obey. The axioms are as follows:

A1. $\Phi(\mathrm{x}_1, \mathrm{x}_2)$ should be defined for all vectors $\mathbf{V_1}$ and $\mathbf{V_2}$ whose elements are non-negative integers.

A2. $\Phi(\mathrm{x}_1, \mathrm{x}_2)$must be non-negative for all $\mathbf{V_1}$ and $\mathbf{V_2}$. (The number of marriage occurring cannot be negative.)

A3. The sum of $\Phi(\mathrm{x}_1, \mathrm{x}_2)$ for all the married males in x_1 must be less than V_2, and the sum of $\Phi(\mathrm{x}_1, \mathrm{x}_2)$ for all the married female in x_2 must be less than V_1. (The number of marriage cannot exceed the number of available female and males.)

A4. The number of marriage should depend heavily on the age of male and female. So, in partitioning male and female components of the population into distinct categories by factors relevant to marriage analysis, at the very last, age must be recognized as an essential factor.

A5. $\Phi(\mathrm{x}_1, \mathrm{x}_2)$ should be a non decreasing function of $\mathrm{V}_1$and V_2, and be strictly increasing for some values of V_1 and V_2. (Increasing availability should not decrease the number of marriages.)

A6. $\Phi(\mathrm{x}_1, \mathrm{x}_2)$ should be a non-increasing (and over some interval strictly decreasing function) of $\mathrm{V}_{x'_1}$ and $\mathrm{V}_{x'_2}$ for $x'_1 \neq x_1$and $x'_2 \neq x_2$. This is McFarland's competition axiom: An extra supply of single men of a certain age should decrease the marriage chances of men at other ages, and similarly for women.

A7. The negative effect on $\Phi(\mathrm{x}_1, \mathrm{x}_2)$of an increase in $V_{x''_1}$ should be greater than the negative effect on $\Phi(\mathrm{x}_1, \mathrm{x}_2)$ of an equivalent increase in $V_{x^{\#}_1}$if x''_1 is closer to x_1 than $x^{\#}_1$ is. Likewise with the sexes interchanged. This is the substitution axiom (or relative competition axiom).

The last three axioms are the most important ones. Actually the models of Pollard and Schoen satisfy McFarland's axioms A1-A6. All fail to comply with axiom 7 (competition is weaker the larger the age difference is between the competing men or women). Axiom 7 means also that the relative competition of men in a given age compared with men in another age group is the same irrespective of the age of the female seeking a mate. But if a female is unsuccessful to find a mate in her first favorite group, she would rather have as second best choice in another age group than her first favorite group.

John Pollard (1993) proposed a two-sex marriage model to resolve this problem. In particular, it would adequately reflect competition and substitution in the marriage market. The following marriage function ("Generalized Harmonic Mean")does seem to have acceptable substitution properties:

$$\Phi(\mathrm{x}_1, \mathrm{x}_2) = k_{x_1 x_2} V_1(x_1) V_2(x_2) / (\sum_{x_1} g_{x_1 x_2} V_1(x_1) + \sum_{x_2} h_{x_1 x_2} V_2(x_2)) \; j = 1, 2$$

the same notation for $\Phi(\mathrm{x}_1, \mathrm{x}_2)$ and $k_{x_1 x_2}$as before

Where the $\{g_{x_1 x_2}\}$ are weights reflecting the relative attractiveness of males aged x_1 to females aged x_2 and the $\{h_{x_1 x_2}\}$ reflect the relative attractiveness of females aged x_2 to males aged x_1. Intuitively, $g_{x_1 x_2} \geq 0$ and $h_{x_1 x_2} \geq 0$.

Here, we need the prime, one prime is sufficient. A small increase in the number of x'_1-year old unmarried males will reduce the number of marriage for males aged x_1. But if the extra supply of unmarried males occurs in the same age group, the number of marriages in this group increases, provided that the coefficients $\{g_{x'_1 x_2}\}$ and $\{h_{x_1 x'_2}\}$ are appropriately scaled. For the competition among females we can apply a similar logic. When the function of the weights $\{g_{x_1 x_2}\}$ for various values of x_2 and x_1 is unimodal and symmetric around some pivotal ages, the model also fulfils the relative competition requirement.

This model has attractive properties, in which genuine competition across ages occurs. Unfortunately, this model is hard to analyze empirically. There are two obstacles for empirical application of this model. One is the large number of age combination coefficients it contains. Some of them are irrelevant, because they occur to odd age combinations, such as a 15 years old man marrying a 60 years old woman. The coefficient of this case can be ignored. But even when we only analyze the most frequent age combinations, the numbers of parameters still are huge. The second obstacle is that the model requires quite detailed data on marriages by age combinations. Pollard made no suggestions whatsoever on this topic.

In 2001, John K. Dagsvik, Helge Brunborg, and Ane Flaatten published a marriage model based on the two-side matching behavior theory and game theory to analyze the marriage markets; see Dagsvik et al (2001). In the marriage market, the total number of marriages depends on the marriage preference. Therefore, under some specific assumptions about the distribution of the marriage preference and the rules of the matching game, the marriage behavior is considered a specific matching game played by the agent. Unfortunately, this model does not satisfy all

McFarland Axioms. In some case, A6 does not hold, and also the authors are unable to prove whether or not A5 and A7 hold.

There are two key elements in Dagsvik's marriage model. One is the deferred acceptance algorithm, which gives a rough approximation to the real marriage game. The deferred acceptance algorithm is outlined as follows.

1. Each man makes an offer to his favorite woman. If he is not rejected by her, he is temporarily engaged until better offers arrive. If he is rejected by the woman, he will move to his next choice making an offer again.
2. Each woman can receive more than one offer. Each woman receiving offers rejects any from unacceptable men, but she marries if the most preferred man is among the group of the new offers.
3. The algorithm stops after any step in which no man is rejected.

The other element is stable matching. Stable matching exists for every marriage market. The argument is as follows.

1. Man A prefers woman B to his partner, but they are not matched to each other.
2. Woman B must be acceptable to man A, so man A must make an offer to woman B before he is engaged.

Since man A was not engaged to women B when the algorithm stopped, he must have been rejected by her. Therefore, woman B is matched to another man whom she likes more than man A. Actually woman B and man A do not block the matching, hence it is called stable.

3.2. The New Two-Sex Model (Juha M. Alho and Nico Keilman 2008)

In this part, we describe the theoretical point of departure and the structure of the marriage model proposed by Alho and Keilman (AK henceforth). For a detailed description of the conceptual framework with the proofs we refer to the recent paper 'A class of Coherent Stochastic Models of Nuptiality', see Alho and Keilman (2008). I will give a brief account here.

The AK model defines a class of individual level stochastic models to describe marriage formation. In specific cases the model can be parameterized in terms of the overall level of nuptiality, the relative quality of spouses by age, and the relative mutual preferences across pairs of ages. Given the relative qualities of the potential spouses, individuals are matched according to marriage probabilities that reflect observed preferences.

Now, in order to represent both the quality of the spouses, and their mutual preferences, we can define a function $\emptyset\ (x_1, x_2) > 0$, for $0 \leq x_1 < \omega, 0 \leq x_2 < \omega$. For identifiability, we assume that this is a probability density, with marginal densities

$$\emptyset_1(x_1) = \int_0^w \emptyset\,(x_1, z)\,dz, \qquad \emptyset_2(x_2) = \int_0^w \emptyset\,(y, x_2)\,dy \tag{1}$$

$\emptyset_1(x_1)$ and $\emptyset_2(x_2)$ will be used to represent the quality of the spouses. As such, they will be fundamental parameters in this model.

We take time to be continuous, and assume that at exact time t, the total marriage intensity of sex j=1, 2, is Λ_j(t)> 0, and the age-specific intensity of marriage is $\Lambda_j(x_j, t)$=$\Lambda_j(t)\emptyset_j(x_j)$. The average marriage intensity is Λ(t) = $(\Lambda_1(t) + \Lambda_2(t))/2$. Primarily, we will consider populations in which the average intensity is fixed, Λ(t) = Λ > 0. This is a fundamental parameter in this model. Note that the marriage intensities for both sexes, and hence also the average intensity are period indicators. We will write Λ_1(t) = (1+ c(t))Λ, so that Λ_2(t) = (1- c(t))Λ. The parameter c(t) reflects the overall imbalance between the sexes in the marriage market.

Here, we also use $V_j(x_j, t)$ to represent the size of the unmarried population of sex j, in ages less than or equal to x_j, at exact time t, for *j*=1, 2. Our population will be finite, V_j(ω, t)$< +\infty$. To ensure coherence, the hazard *h*(t) of a new marriage in the population must be the same for both agents. Using Stieltjes integral notation, we get that

$$h(t) = \int_0^w \Lambda_1\,(x_1, t) dV_1(x_1, t) = \int_0^w \Lambda_2\,(x_2, t) dV_2(x_2, t) \tag{2}$$

Define

$$W_j(t) = \int_0^w \emptyset_j\,(x_j) dV_j(x_j, t)\ ,\ j = 1, 2 \qquad (3)$$

Since $\Lambda_1(x_1, t)$= (1+ c(t))$\Lambda\emptyset_1(x_1)$, and $\Lambda_2(x_2, t)$= (1+ c(t))$\Lambda\emptyset_2(x_2)$, the second equality of (2) implies that (1+c(t))W_1(t) = (1- c(t))W_2(t). Therefore, we have that

$$c(t) = (W_2(t) - W_1(t))/(W_1(t) + W_2(t))$$

Taking sex j = 1 as a starting point, and substituting (4) into (2), we get that h (t) =Λ H (t), where

$$H(t) = \frac{2W_1(t)W_2(t)}{W_1(t)+W_2(t)}$$

In other words, the hazard of a new marriage is the product of the average intensity and the harmonic mean of the quality weighted population sizes.

So far, I have mainly presented the model aspects that refer to either men or women. However, the AK-model also includes a feature which describes the interaction between the sexes. Below I shall only give an intuitive description. The details are complicated (see AK, Section 2.2), and an empirical analysis of the interaction patterns between men and women who marry is outside the scope of this thesis. This must be part of a future research agenda.

As stated above, mutual preferences of men and women are given by the joint probability density $\emptyset(x_1, x_2) > 0$, for $0 \leq x_1 < \omega, 0 \leq x_2 < \omega$. Using the theory of log linear models, this density can be decomposed into four types of multiplicative effects: an overall effect, an age-specific main effect for men, and age-specific main effect for women, and an interaction effect specific for each age combination. Given an empirically specified density $\emptyset(x_1, x_2)$ for discrete ages, the decomposition can be computed using the Iterative Proportional Fitting algorithm (Deming 1943). In the AK-model, the key assumption concerning $\emptyset(x_1, x_2)$ is that the interaction effect is independent of time. However, the overall effect and the main effects for men and women may change as a result of changes in the numbers of potential spouses over time.

I now turn to the estimation of the quality parameters for men and women.

In order to estimate the quality parameters, we can use the method of Maximum Likelihood Estimation. We assume piecewise constant intensities, and write $\emptyset_j(x_j) = \emptyset_{ji}$ for $i \leq x_j < i + 1, i = 0, \ldots, \omega - 1$. Let the population in age i at exact time $0 < t < 1$ be V_{jit}. It follows from (2) that if a marriage occurs at exact time t, the probability that it occurs to some individual in age $i = 1, \ldots, \omega - 1$, is proportional to $\emptyset_{ji} V_{ijt.}$ We can approximate $V_{jit} \approx V_{ji}$, where V_{ji} is the number of person years lived during $[0,1]$. Suppose there are M marriages during the whole period. Under a multinomial model the expected number of marriages in age $i = 0, \ldots, \omega - 1$ is

$$E\left[M_{ij}\middle|M\right] = M\emptyset_{ij}V_{ij} / \sum_{k=0}^{\omega-1} \emptyset_{jk}V_{jk}$$

Then, the approximate MLE's of the qualities are simply

$$\emptyset_{ji} = \frac{M_{ji}}{V_{ji}} / \sum_{k=0}^{\omega-1} \frac{M_{jk}}{V_{jk}}, \quad j = 1, 2.$$

In other words, these are the usual occurrence exposure rates normalized to sum to one. Based on Maximum Likelihood estimates of the quality parameters, an ML-estimate of the average intensity Λ may be computed as follows:

1. Compute W_1 and W_2 using expression (3)
2. Compute H as the harmonic mean of W_1 and W_2
3. Compute Λ as M/H

This is the procedure proposed in the original paper (see AK, Sections 3.2 and 3.3). However, a simplified procedure will be outlined in Section 4.

As discussed in AK, this marriage model above is consistent with a large number of matching algorithms.

In Table 1, I list the nuptiality models mentioned above and show whether they meet the McFarland requirements.

Table 1. Performance and characteristics of recent nuptiality models: extent to which they meet theoretical requirements (Nico Keilman 1994).

	Requirements						
	A1	A2	A3	A4	A5	A6	A7
(1) Panmictic circles, Henry (1968, 1972)	yes	yes	yes	yes	no	no	no
(2) Iterative adjustments, McFarland (1975)	yes	yes	yes	yes	yes	?	?
(3) Harmonic means model, Schoen (1977)	yes	yes	yes	yes	yes	yes	no
(4) Generalized harmonic means, Pollard (1993)	yes	yes	yes	yes	yes	yes	no
(5) A Behavioral Two-Sex Marriage Model Dagsvik, Brunborg and Flaatten (2001)	yes	yes	yes	yes	?	no	?
(6) A class of Coherent Stochastic Models of Nuptiality, Alho and Keilman (2008)	yes	yes	yes	yes	yes	yes	yes

4 Data and analysis

For each of the years 1974-2002 the following data for the case of Norway were available. 1. Numbers of marriages broken down by year of birth and previous marital status of both partners; 2. Numbers of men and numbers of women on 1 January broken down by year of birth and marital status. I have calculated Maximum Likelihood estimates concerning the marriage intensities, the marriage rates, the gender quality parameters and the gender imbalance term (c term). Moreover, my thesis will compare some of the results with those for the classical one-sex marriage model. I analyze the Norwegian data for the years 1974, 1980, 1985, 1990, 1995, 2000, 2001, and 2002. For the last few years, I analyzed the data for single years, in order to see sufficiently accurate details for the most recent years.

For each year, I analyze two data sets independently: one for first-marriages and one for all marriages. Data for first marriages are restricted to those cases where both partners marry for the first time. Data for all marriages include both first marriages and remarriages. Note also that I restrict the analysis to person's age under 60. When a person's age is over 60, the event of marriage happened is very rare.

An analysis of marriage rates requires data on the exposure time (the number of person years lived) of unmarried population during a particular year. However our data set lacks information of this kind. Therefore, I approximate the exposure time by the mid-year unmarried population, computed as the unmarried population in the beginning of the year minus half of the number of marriages. In this procedure we disregard the effects of mortality and migration. Mortality in Norway is not very important for ages below sixty. Migration may have some effect: the immigration surplus observed for most of the period implies that denominators are slightly too low, and hence marriage rates are a bit too high.

Now, I propose some relevant details and the key formulas for the classical one-sex marriage model to compute the marriage indicators.

Explicitly stated, the one-sex marriage model only takes into account the single sex and mortality in order to give an interpretation of marriage behavior. This type of model indicates the pace at which a group of single persons is decreased form one age to the next one as a consequence of marriage and death, and it also gives the probability of a single person marrying at each year of age according to the current marriage and mortality rates. The one-sex model gives information on the average age at marriage, a measure of the proportion of single persons who remain single at each age, and the proportion of persons who will eventually marry. Details are given in demography text books, for instance Shryock and Siegel (1976) or Preston (2001) et al.

In my application of the one-sex model I have ignored mortality, for reasons stated above. Also, I have restricted myself to first marriages. One-sex models that include both first marriages and remarriages do exist, but these require complicated matrix expressions, and as such they fall outside the scope of my thesis project.

A one-sex first marriage model summarizes the marriage behavior of a hypothetical cohort, which experiences first marriage in conformity with a set of age-specific marriage rates. The first-marriage rate for sex *j* at age i can be written as

$$m_{ji} = \frac{M_{ji}}{V_{ji}}, j = 1{,}2; \;\; i = 1, 2, \dots, \omega - 1$$

The marriage probability is a very important parameter in the one-sex marriage model. The marriage probability represents the chance that a marriage will occur over a specified period of time to a person of a given age, sex, and marital status at the beginning of the period. Here, I typically assume that 1000000 unmarried persons at age 15 (also called radix) will be exposed to the risk of a first marriage. The marriage intensity will be the total number of marriages as predicted by the model, divided by the radix.

The marriage rates can be converted into marriage probabilities for persons at exact years of age using the formula shown below:

$$n_{ji} = \frac{2m_{ji}}{2 + m_{ji}}$$

In the one sex model, I denote the number of marriages in age i for sex j as M_{ji}, and define the size of the unmarried population at in aged i in the model as l_{ji}. Then the unmarried population is computed as $l_{j,i+1} = l_{ji}(1 - n_{ji}), j = 1,2; i = 1,2, \dots, \omega - 1$. The radix is $l_{j,15} = 100000$. Marriages at each age are computed as $M_{ji} = l_{ji}.n_{ji}$.

The marriage intensity for single sex is may be computed using the following formula

$$\Lambda_j = \frac{\sum M_{ji}}{1000000} \; j = 1,2$$

where the sum is taken over all ages.

Thus, in the one-sex model, the marriage intensity is the total number of marriages as a share of the radix. Similar to the case of the two-sex model, the intensity Λ_j of the one-sex model is a period indicator.

I now turn to the two-sex model. Nico Keilman found a way to simplify the calculations of Λ in the two sex model. In this convenient procedure we can skip to compute W_1 and W_2. The key features of this short-cut method are described below.

Using the formula of the marriage rate, cited in the preceding section, would lead to $\emptyset_j(x_j)$ as follows:

$$\emptyset_j(x_j) = \frac{m_j(x_j)}{\sum m_j(x_j)} = \frac{M_j(x_j)/\,V_{ji}}{\sum M_j(x_j)\,/V_{ji}}$$

Now we will develop a new formula in term of $W_j(t)$, $j = 1,2$

$$W_1 = \sum \emptyset_1(x_1) \times V_{ji} = \sum \frac{M_1(x_1)/\,V_{ji}}{\sum M_1(x_1)\,/V_{ji}} \times V_{ji} = \frac{\sum M_1\,(x_1)}{\sum M_j(x_j)\,/V_{ji}}$$

The numerator of the right-most term in that expression is equal to M_1 which represents the total number of new marriages for males in the year.

Therefore, the desired value of W_1 is calculated by the use of

$$W_1 = \frac{M_1}{\sum M_1(x_1)\,/V_{ji}}$$

The method of calculating the W_2 for females is straightforward by changing the subscript as 2, which can be represented as

$$W_2 = \frac{M_2}{\sum M_2(x_2)\,/V_{ji}}$$

where $M_2 = M_1 = M$. For convenience, I will write $\beta_1$denote the term of $\sum M_1(x_1)\,/V_{ji}$, similarly, $\beta_2$shows the term of $\sum M(x_2)\,/V_{ji}$.

Concerning these equations mentioned above, $\mathrm{H}(t)$ may be expressed as

$$\mathrm{H}(t) = \frac{2M^2}{\beta_1 \times \beta_2} \times \frac{1}{(M/\beta_1) + (M/\beta_2)} = 2M/(\beta_1 + \beta_2)$$

The basic formula for the marriage intensity across two sexes is

$$\Lambda = \frac{M}{H(t)} = \frac{M}{2M} \times (\beta_1 + \beta_2) = \frac{\beta_1 + \beta_2}{2}$$

The relevant male and female intensities may be calculated after having computed the term $\mathrm{c}(t)$.

$$c(t) = \frac{W_2 - W_1}{W_1 + W_2} = \frac{\frac{M}{\beta_2} - \frac{M}{\beta_1}}{\frac{M}{\beta_2} + \frac{M}{\beta_1}} = (\beta_1 - \beta_2)/(\beta_1 + \beta_2)$$

Thus, the derivation of Λ_1 and Λ_2 can be obtained by working backwards from all the equations in this section.

$$\Lambda_1 = [1 + c(t)] \times \Lambda = \beta_1 \qquad \Lambda_2 = [1 + c(t)] \times \Lambda = \beta_2$$

This method is useful, for instance, when manual calculation is necessary. As in the one sex model (i.e. the first -marriage table), in the two-sex model, the formula measures the intensity which reflects the empirical marriage rates across the whole age interval in each sex.

In the empirical analysis of the two-sex marriage model and the one-sex marriage model I shall analyze the following:

- Trends in the overall level of marriage intensities, gender quality parameters, and c term reveal developments in the marriage behavior in Norway irrespective of age and sex in the period of 1974-2002.

- Trends in differences in intensities between the two marriage models.

- Differences of the intensities between the sexes.

- Differences of the marriage rates and gender quality parameters between the age groups.

The above features are the subject matters of the intensity, marriage rate and gender quality parameters that measure the principal features of marriage formation. The most commonly used of these are easily understood and calculated. In the rest of this chapter, I will introduce them to provide a step-by-step approach to outline the effective explanation, together with an example by adopting Norwegian data in 1985, 1990 and 2002 and figures of their applications in 1985 and 1990. Herein, making a notice, I will use the example by selecting the data 1985, 1990 and 2002 in order to show the conceptually illustrations in part of sections. The graphs attached in my thesis show that the main trends are always consistent over the period.

4.1. Trends of Norwegian Data in the period 1974-2002

In view of the aggregate Norwegian data trends concerning nuptiality in the period 1974-2002, now we will discuss the results for the two-sex marriage model.

Figures 1 and 2 summarize the trends in intensities for selected years in the period 1974 to 2002.

Figure 1

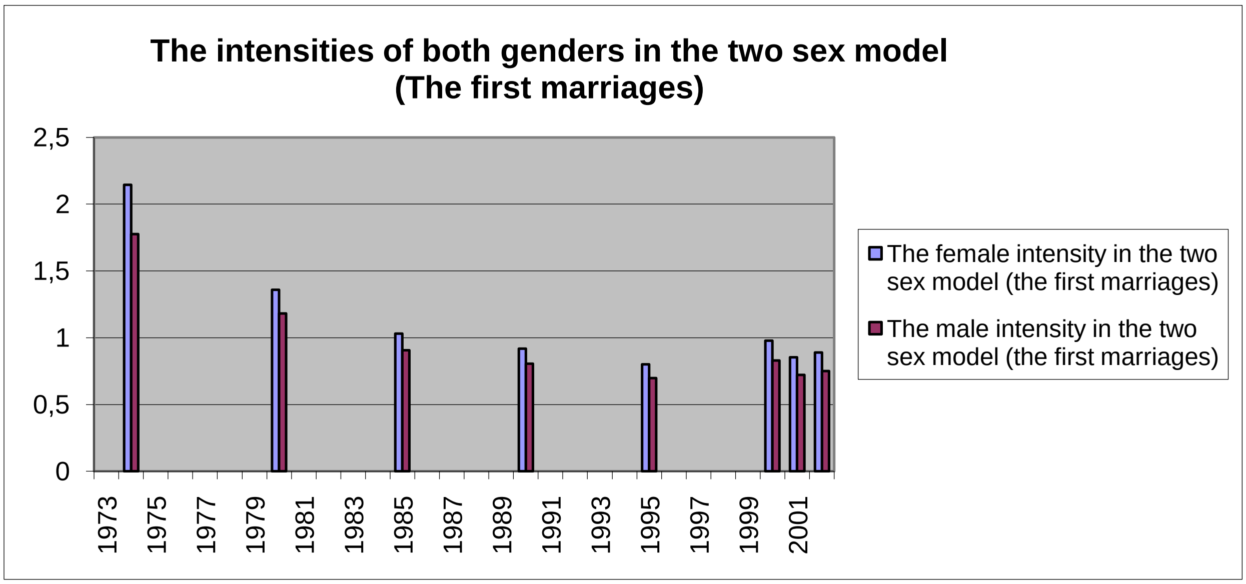
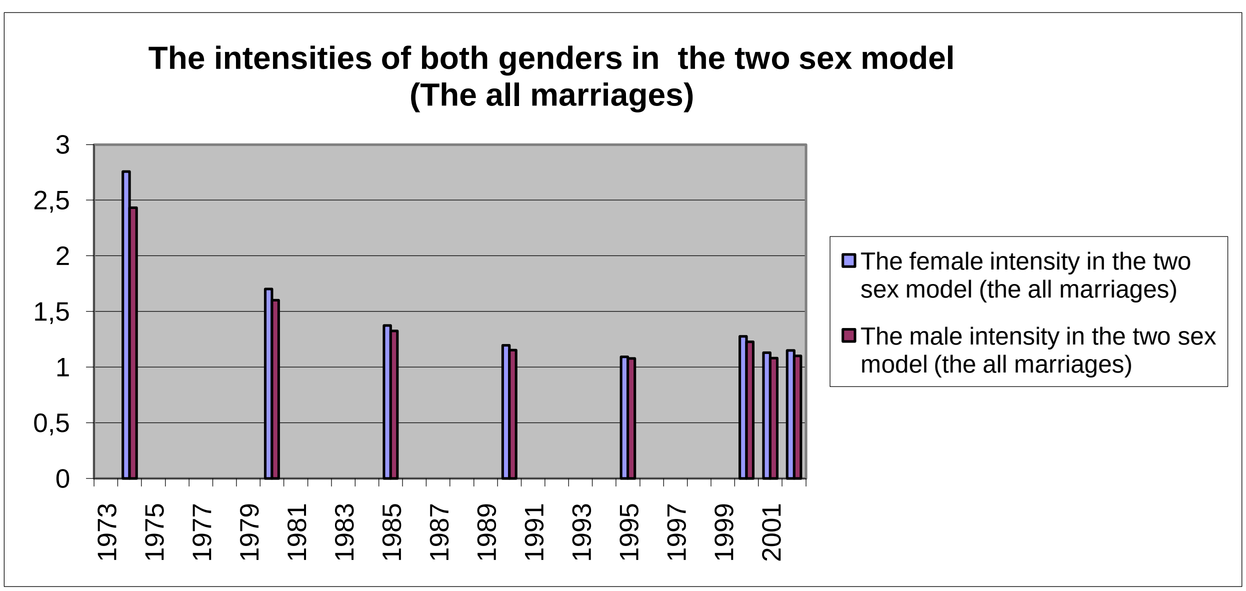


Figure 2



In Norwegian data from 1974-2002, the decrease in marriage intensities came to an end around 1995; after that, the intensities seem to fluctuate a little. The irregular pattern in the last few years is possibly due to randomness.

In Norway, we observe a strong variation in marriage intensities over the years. The highest estimate of the intensities occurs in 1974, and the lowest one appears in 1995.

As we know, the marriage intensity is equal to the sum of age-specific marriage rates. Figures 3 and 4 give more details for explaining the declined marriage rates from 1974 to 1995. I found that the age patterns of male and female rates from 1974 to 1995 always shift to the right, while the top of the curves fall markedly. This explains why the intensities for men and women decline over the years. And also the graph shows that almost all of the decreases occurred in ages over 30 years. For instance, for first-marriages in 1995, the pattern for males increases first until the age around 29-30 from the youngest age 17, and for later ages it decreases again.

Figure 3

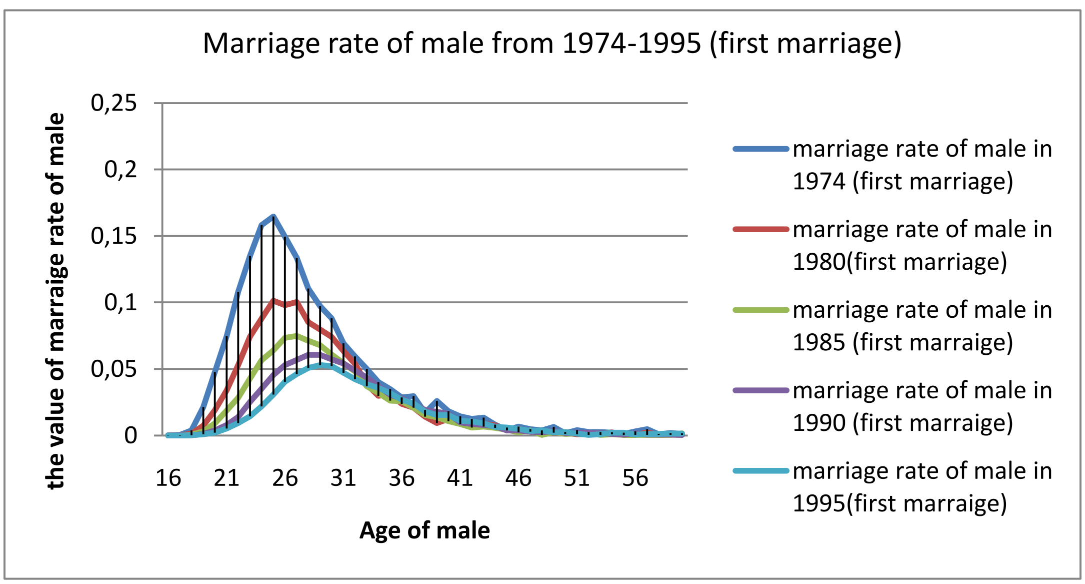
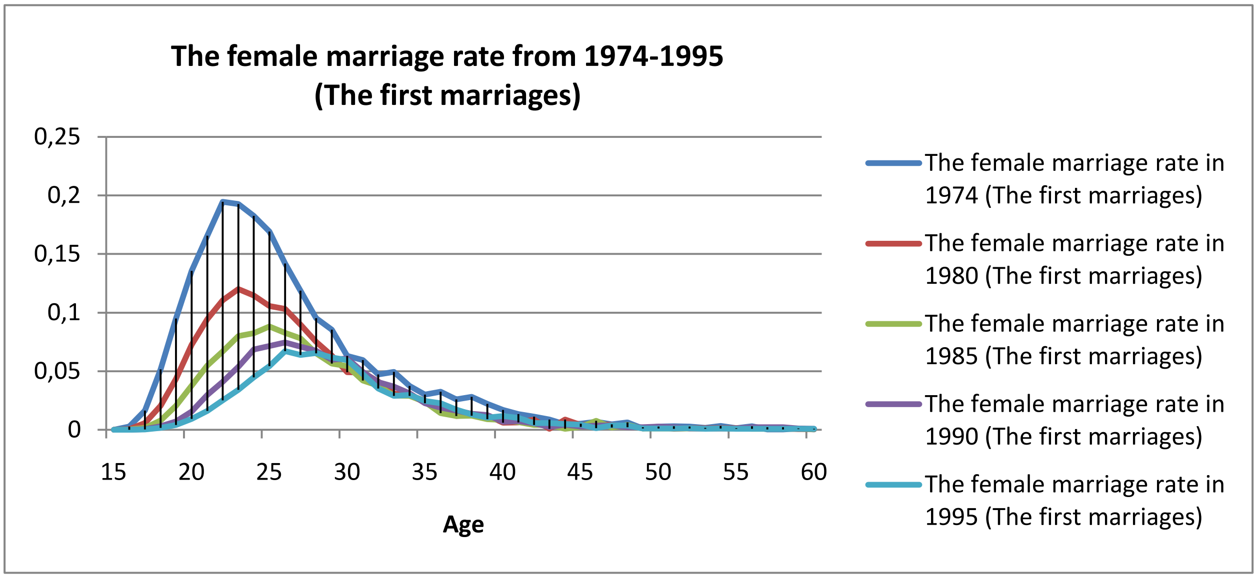


Figure 4



Moreover, the other indicators which are the overall marriage intensity Λ and Ø show very similar outcomes compared to the intensities of genders.

Actually, the intensities of the two sexes are used to compute the overall marriage intensity according to the formula:

$$\Lambda = (\Lambda_1 + \Lambda_2)/2$$

By this, we say that the overall marriage intensity is the arithmetic mean of the overall male and female marriage intensities. Figures 5 and 6 plot the trends of overall marriage intensity from 1974 to 2002; the patterns in those two data sets are almost the same. These two trends are consistent with the trends in male and female marriage intensities in Figures 1 and 2.

The trends of age-specific quality parameters for males from 1974 to 2002 are shown in Figures 7 and 8. For each age, Ø is estimated as the marriage rate as proportion of the sum of all age-specific marriage rates. Hence, they sum to one. The figures show that men over 30 years of age became more attractive over the years. For women, the patterns are similar, and these are not shown here.

Figures 3 to 8 show that all of the curves move to the right following the selected years. Thus the general patterns of indicators trends in the graph representations are very similar. Marriage has become less popular (Figures 5 and 6), those who marry do so at higher ages (Figure 3 and 4), and potential marriage partners below 30 are less attractive than older ones (see Figures 7 and 8 for men). This is explained by the increased popularity of cohabitation in Norway in the 1970s, 1980s, and 1990s. Data on cohabitation show that the proportion of cohabitants compared to all those who live in a relationship has not increased in recent years; see, for example http://www.ssb.no/samboer_en/.

Figure 5

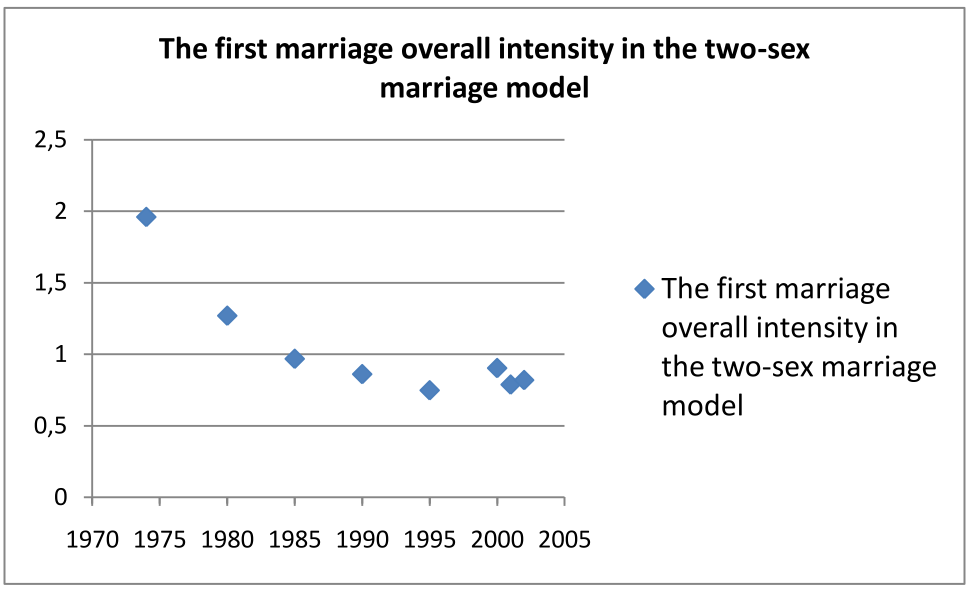


Figure 6

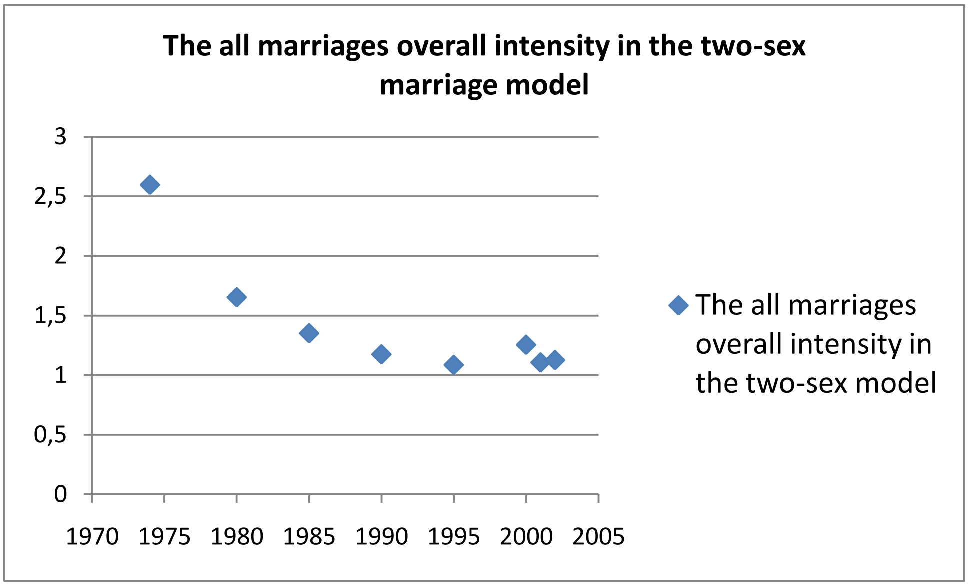


Figure 7

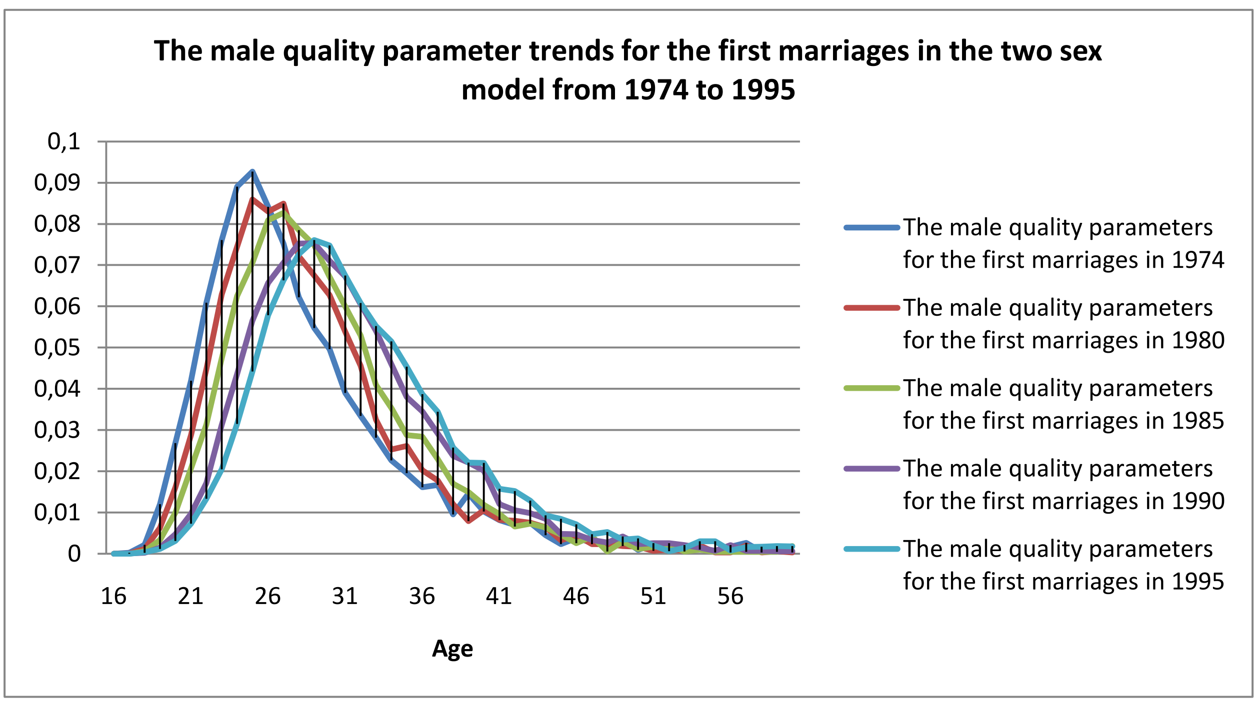
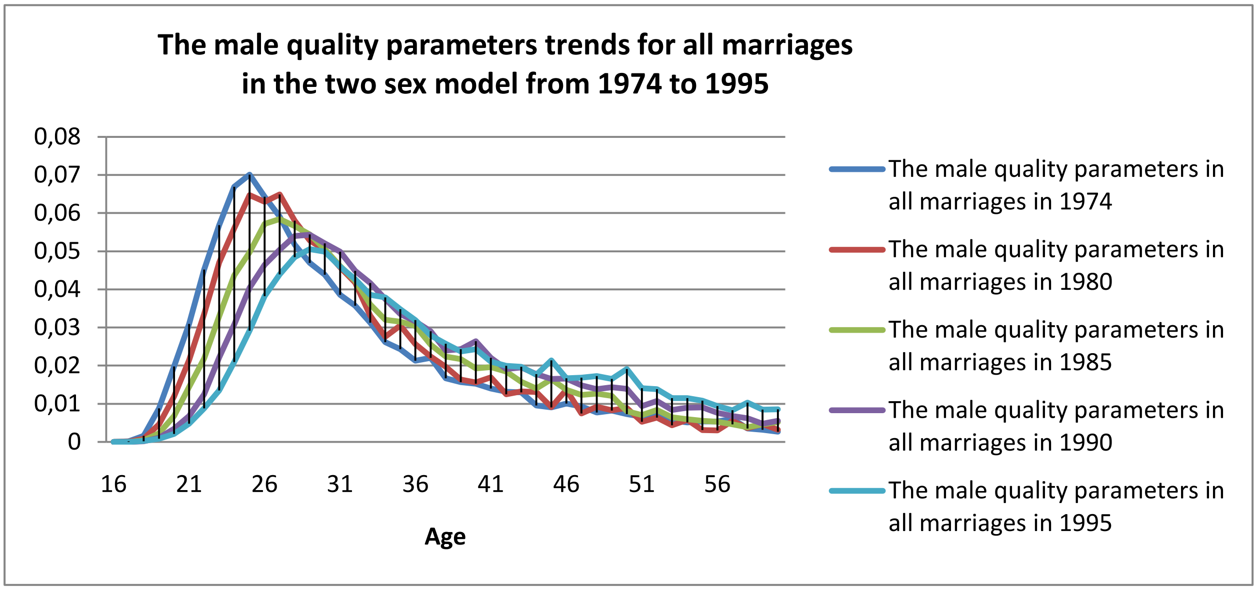


Figure 8



4.2. The Two-Sex Model versus the One-Sex Model

In this section, I will outline how the results of the two-sex model differ from those of the traditional one sex version.

Figures 9 and 10 plot the trends of marriage intensities for women and men estimated for the two models. We observe very large differences in the beginning of the period, but much smaller ones towards the end.

This trend is explained by the ways the intensities are computed. For the two-sex model the intensity is the sum of the age-specific marriage rates. Each rate has a value between zero and two (when everyone in a certain age group marries, the number of marriages equals the initial population, while the mid-year population is half the initial population; hence the marriage rate is equal to two in this extreme case), which means that the sum can very well exceed one. On the other hand, the intensity for the one-sex model is the share who ever marry. This share can not exceed one, by definition. In the 1970s marriage was almost universal, as witnessed by the one-sex intensity which is almost one. Marriage rates were high at many ages, as was their sum. By the 1990s, rates were much lower because of the decreased popularity of marriage, and the sum fell even below one. The one-sex model indicates that only half of the population would eventually marry, if the rates of those years would remain constant.

Figure 9

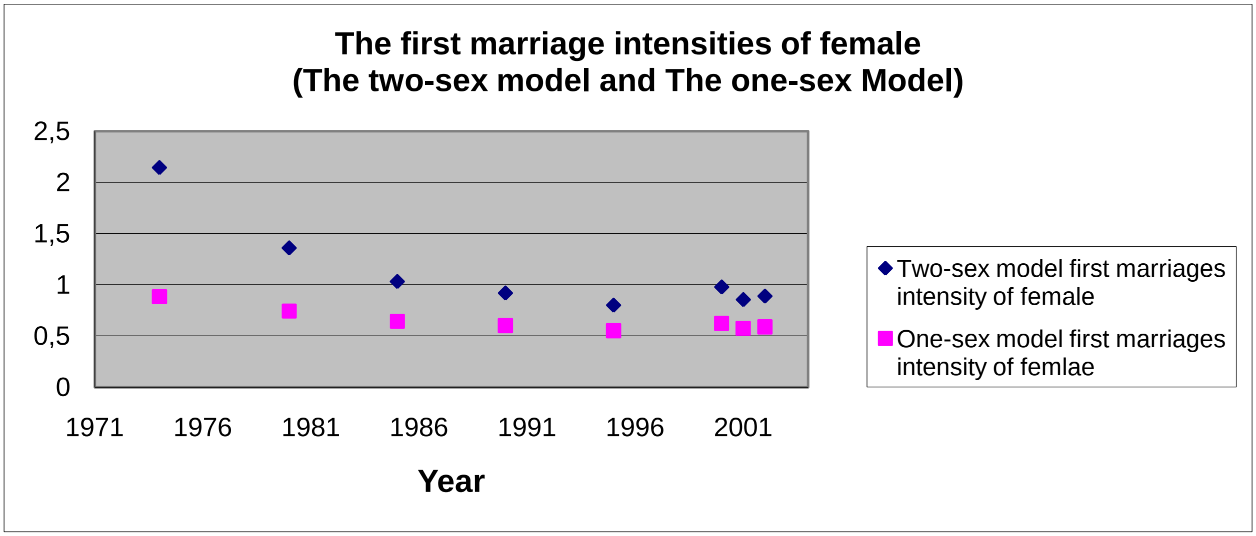
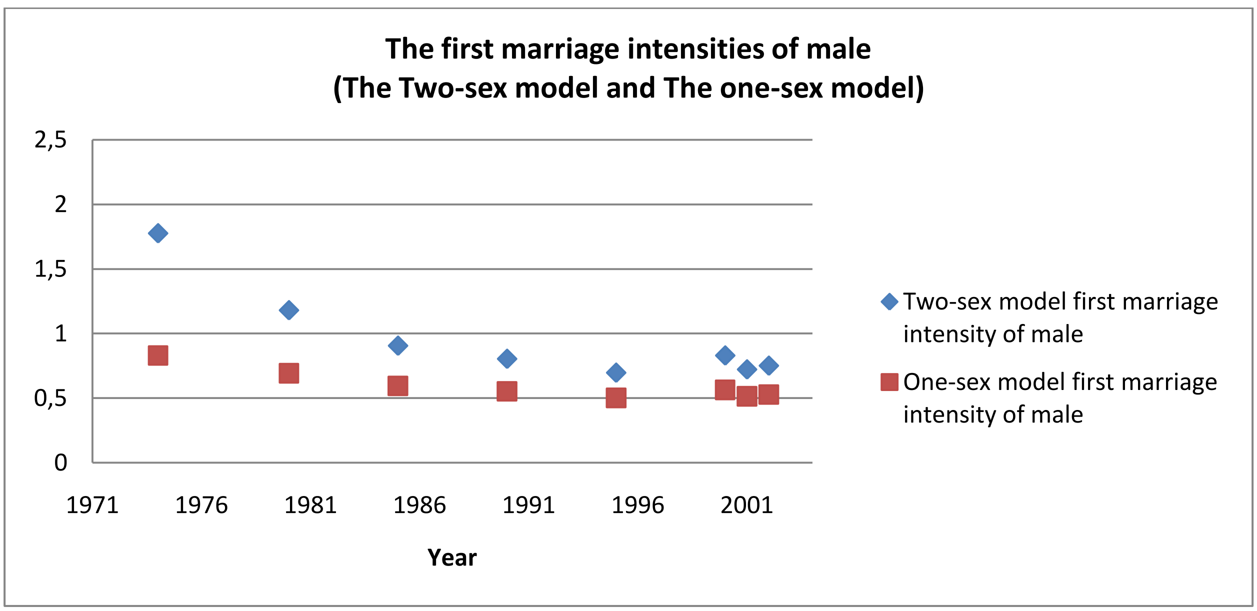


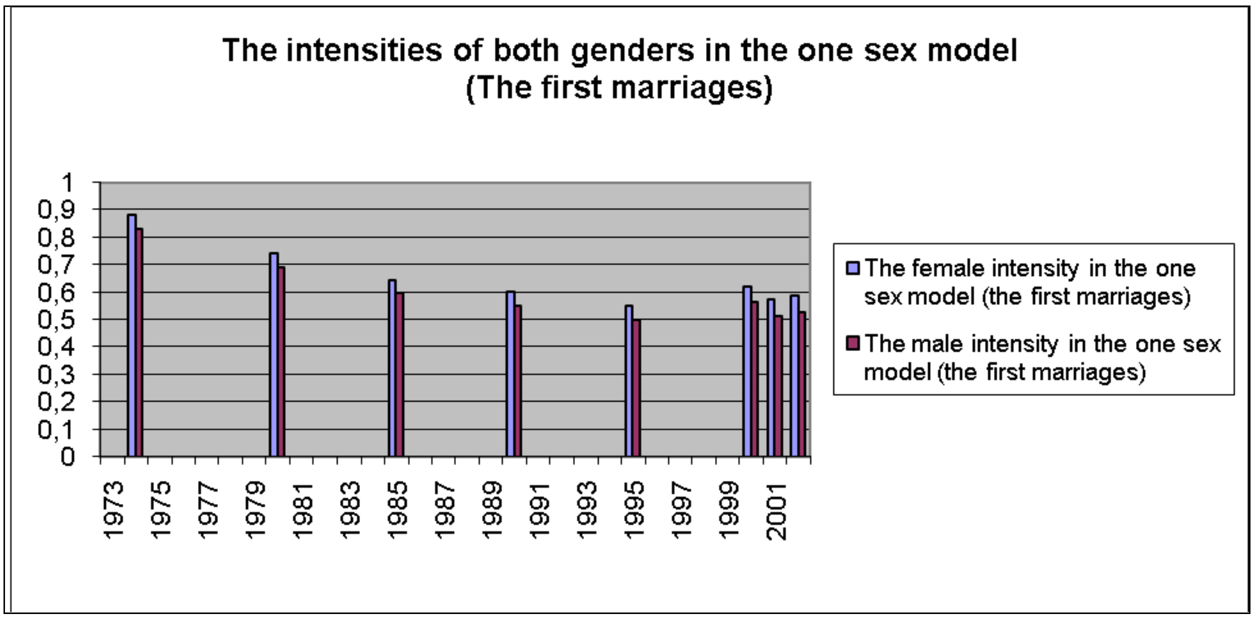
Figure 10



4.3. The trends of male marriage versus the trends of female marriage

Figures 1, 2, and 11 plot the trends of marriage intensities for men and women computed for first marriages and all marriages by using two marriage models. The precise numerical values of male and female intensities are given in Tables 3 and 4 (see page 38).

Figure 11



From Figures 1 and 2 we observe the variations in marriage intensities. I found that female intensities are consistently higher than male intensities in every year. Since one woman typically marries one man, this outcome implies that there are fewer available women than men. When I analyzed the Norwegian data for 2002, I found that the number of never-married men (66059) is higher than the corresponding number for women (510136), and the number of unmarried men (863701) is still higher than that for women (821354).

Actually, the number of available female is lower than the number of male in the restricted age groups below 30 for all estimated years (most marriages occur before age 30.). Intuitively, increasing the number of available spouses will reduce the marriage intensity, but will increase the intensity of marriage of the other sex.

In Norway, the overall the gaps in both gender intensities are clearly visible. I found that female intensities are higher than male intensities in the first marriage data. But they are closer to each other when we analyze the all marriages data in the same year.

Figures 12 and 13 represent the trends in the parameter c across the years 1974 to 2002. In these two pictures, c is always greater than zero. By this, it gives the explanation of gender marriage intensity from another point of view. According to the formula in section 3.2

$$\Lambda_j(\mathrm{t}) = (1 - \mathrm{c}\,(\mathrm{t}))\,\Lambda \quad j\text{=1, 2}$$

c(t) describes imbalances between the sexes which can affect the gender marriage intensity. When c(t) is zero, there are no imbalances between the sexed. c(t) >0 means larger intensities for women than for men relative to the overall intensity. In Figure 12, the imbalances become a bit smaller in years between 1974 and 1985 and they become a bit higher to 2002.

In the all marriage model, the number of marriages and the number of available spouses are higher than in the first marriage model because higher order marriage spouses are added. The two figures show that the imbalances between women and men are smaller in the all marriage model than in the first marriage model. Intuitively, this is plausible, since imbalance for never-married marriage candidates are reduced when partners who have already been married before become available. Yet the time patterns in these two pictures are very similar.

Figure 12

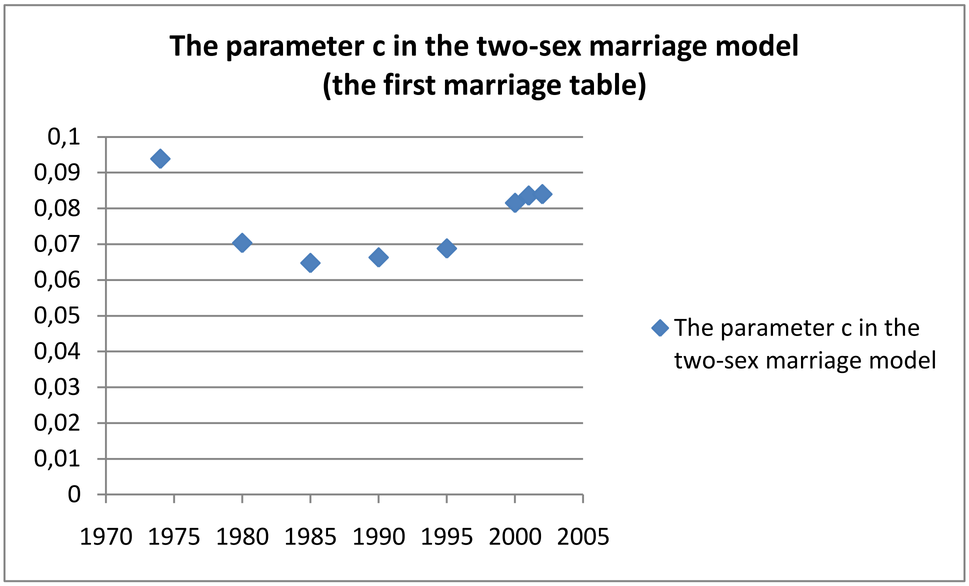
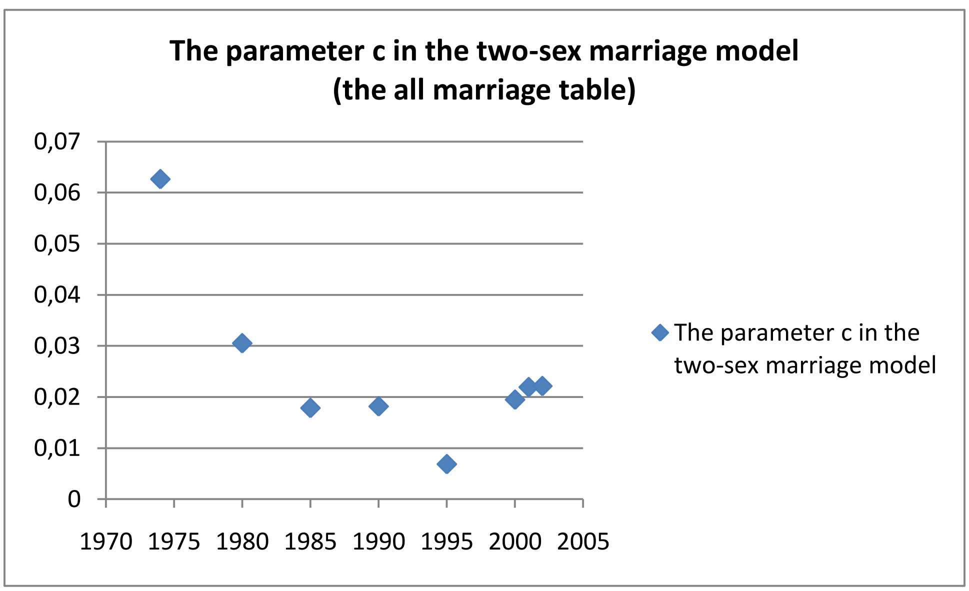


Figure 13



From the view of realism, most marriages normally occur in some typical age groups for male and female, and also it is very rare for some extreme age combinations, such as a very young bridegroom and an old bride. The following observation illustrates this argument. Table 2 gives a summary view of all age combinations for the two genders using Norwegian data for 2002.

Table 2. Marriages by Age of Bride and Bridegroom, Norway, 2002

Age of Bride	Age of Bridegroom										Total
	15-19	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60+	
15-19	10	100	37	6	4	2	0	1	0	0	160
20-24	7	779	1177	341	78	15	8	4	1	0	2410
25-29	1	172	3029	2441	576	106	34	12	6	0	6377
30-34	0	21	616	2479	1328	402	130	42	12	4	5034
35-39	0	10	69	436	912	611	240	109	29	9	2425
40-44	0	2	15	61	214	393	315	177	76	13	1266
45-49	0	0	5	10	49	106	266	240	126	33	835
50-54	0	0	1	1	8	39	85	196	150	67	547
55-59	0	0	0	0	0	3	17	39	105	88	252
60+	0	0	0	0	1	1	4	11	29	116	162
Total	18	1084	4949	5775	3170	1678	1099	831	534	330	19468

According to Table 2, numbers of marriages are highest when the bride is in the same age group as the bridegroom, or in the next younger age group. This confirms what one observes in every-day life: in many marriages the bride is younger than the bridegroom.

Figures 14 and 15 give a summary view of the marriage rates for two genders in the two sex model. All distributions are skewed to the right both in the all marriages data set and for first marriages. Note also that male marriage rates are lower than female marriage rates at prime marriage ages. This is consistent with the pattern in Figure 2, which shows higher female marriage intensities from 1974 to 2002.

Figure 14

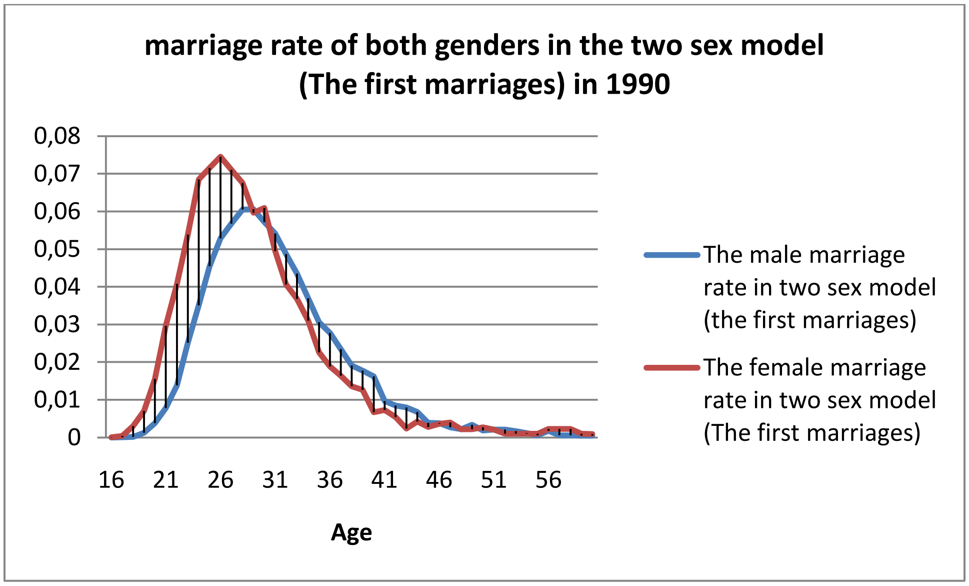
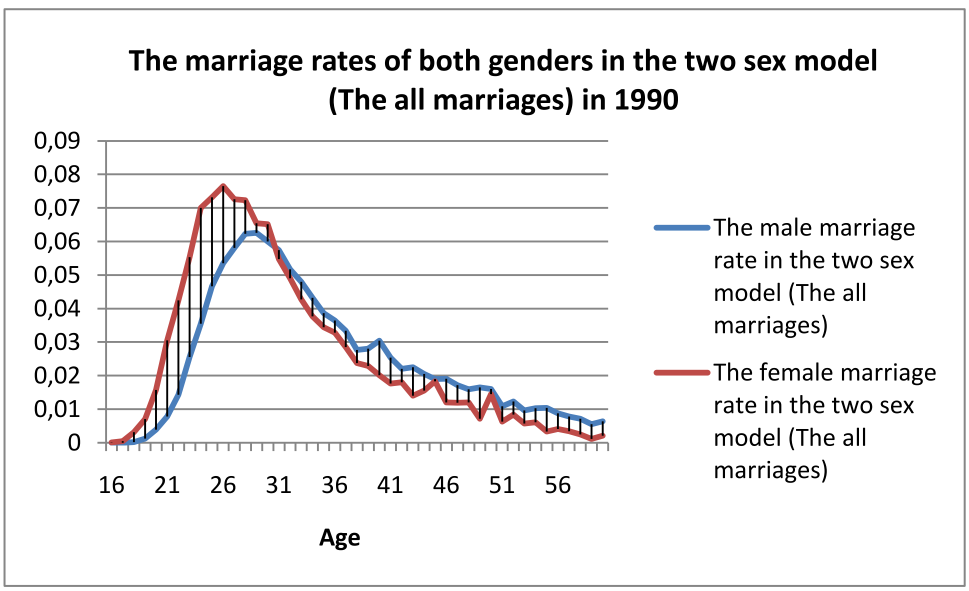


Figure 15



The two-sex marriage model can examine the relative quality of the potential spouses and the mutual relative attraction of spouses in different age groups. Ø is the indicator used to represent the quality of the potential spouse as noted above. The sum of Ø over all ages is equal to one. The calculation is derived by Maximum likelihood Estimation based on the marriage rates, Ø and marriage intensity together. Intuitively, an individual with a high marriage intensity would have marriage rate and Ø value at a high level.

Figures 16 and 17, show the estimates of female and male quality parameters in 1990. For men, the highest value of Ø appears around age 31, for women a few years earlier. The curves reflect the age patterns of the marriage rates for both sexes. Figures 7 and 8, show that the curves for the quality of men shifted to the right during the years 1974 to 1995. The “quality” of young men declined, while that of men older than 30 increased. Note that this applies to a formal marriage. In many cases the couple has been living together in a consensual union for a number of years.

Although Ø, marriage intensity, marriage rate and c-term are different indicators in the two-sex model, the patterns and trends are always consistent.

Figure 16

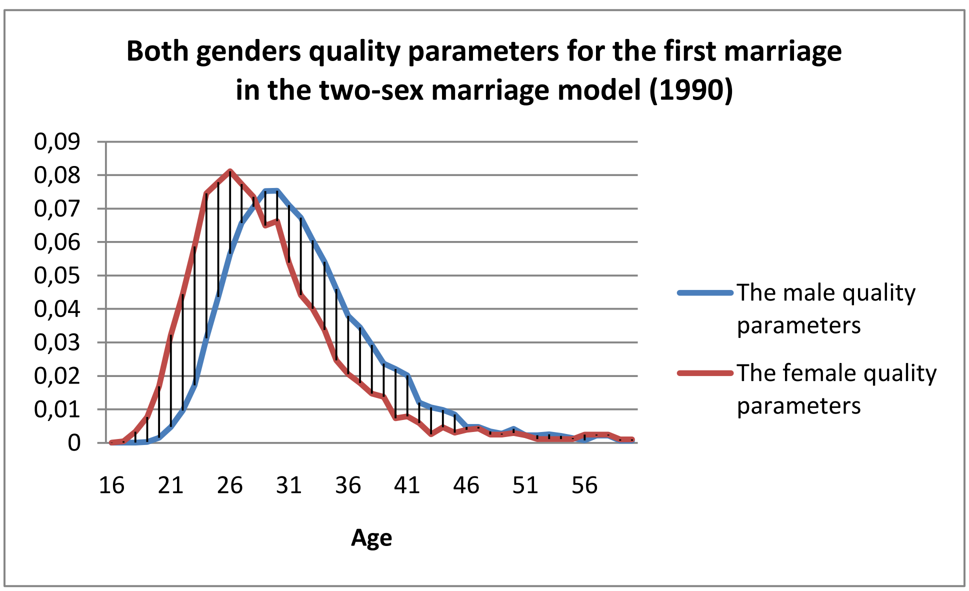
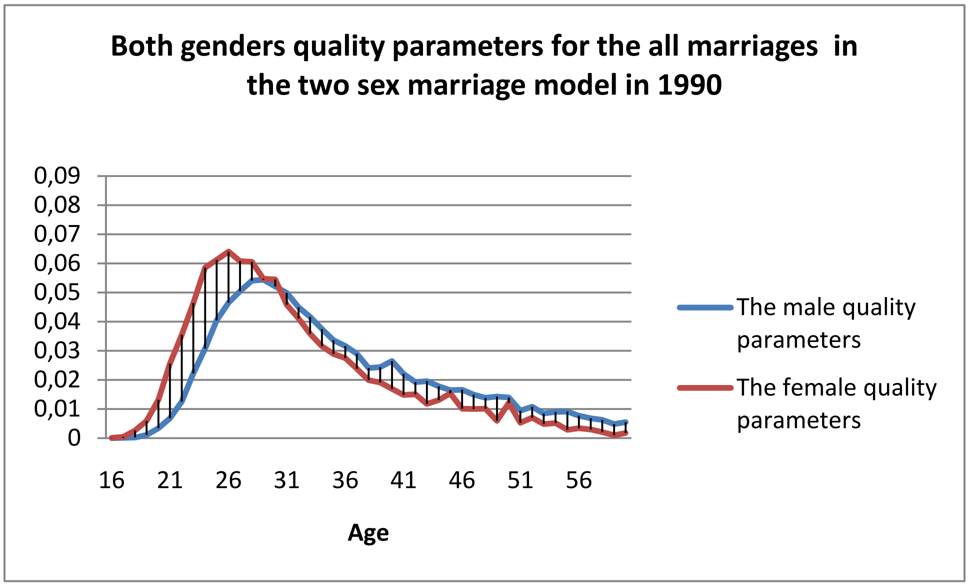


Figure 17



4.4. The trends for first marriage versus the trends for all marriages

Figures 18 and 19 summarize the differences in first marriage intensities compared to those for all marriages for both spouses from 1974 to 2002. The results show that the intensity for all marriages is higher than that for a first marriage, as one would expect: a person can have a first marriage only once, but he/she can remarry a number of times. In practice, the differences between the two types of intensities are not very large. More importantly, the differences are larger for men than for women. This agrees with the fact that men remarry more often than women do.

Figure 18

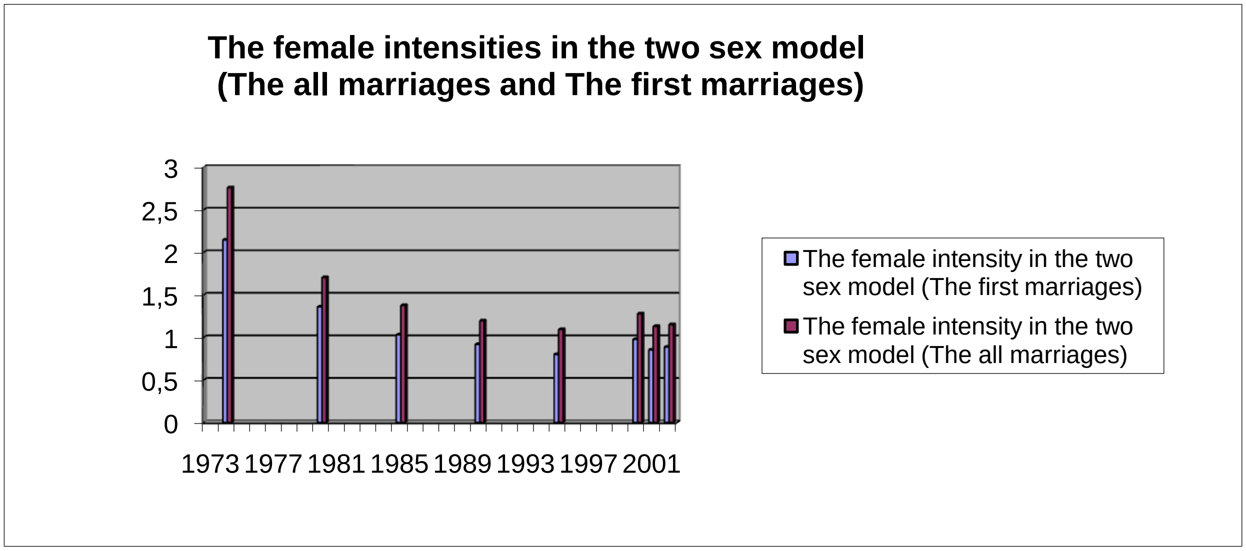


Figure 19

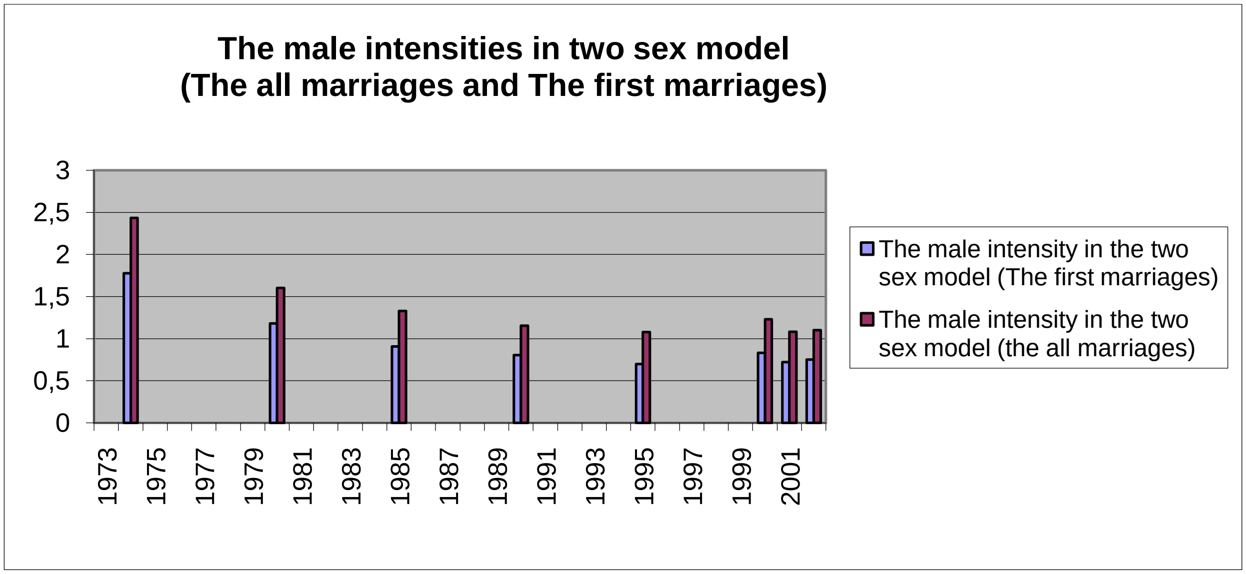


Figure 20

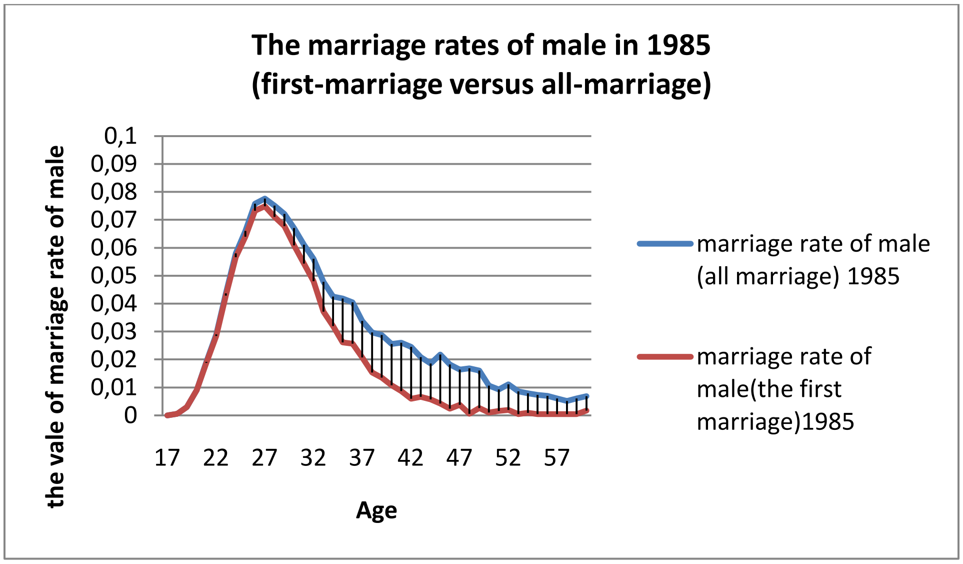
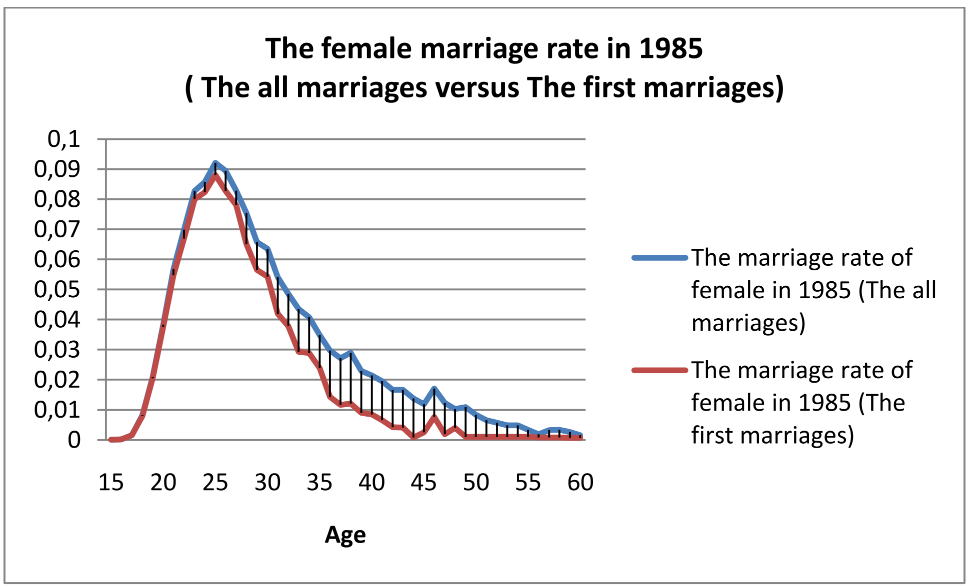


Figure 21



Figures 20 and 21 show age-specific rates for first marriage and for all marriages. At young ages, a marriage is almost always a first marriage and thus the two curves coincide. At higher ages, remarriages occur increasingly often and age-specific rates for all marriages are consistently higher than those for first marriages. This explains why first marriage intensities are lower than those for all marriages.

Figure 22

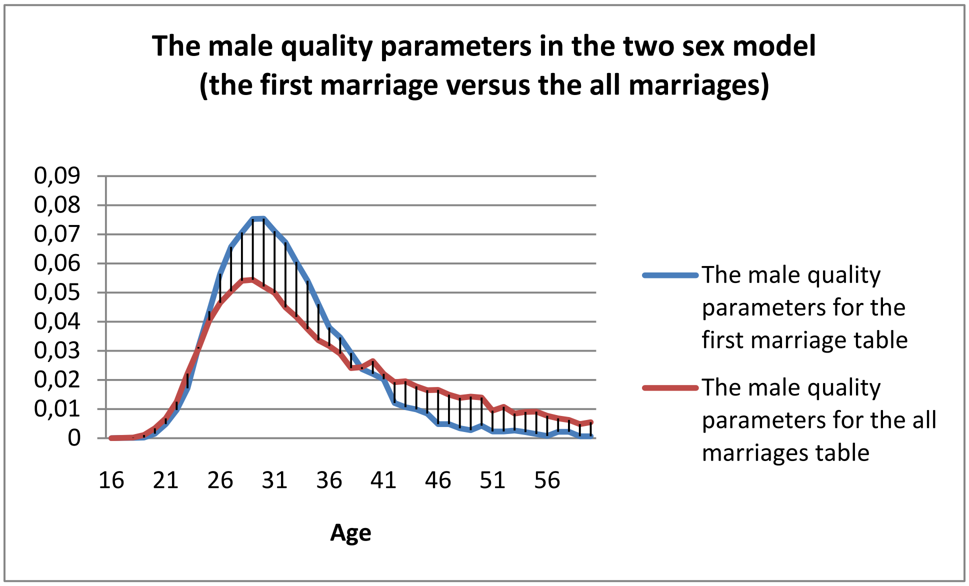
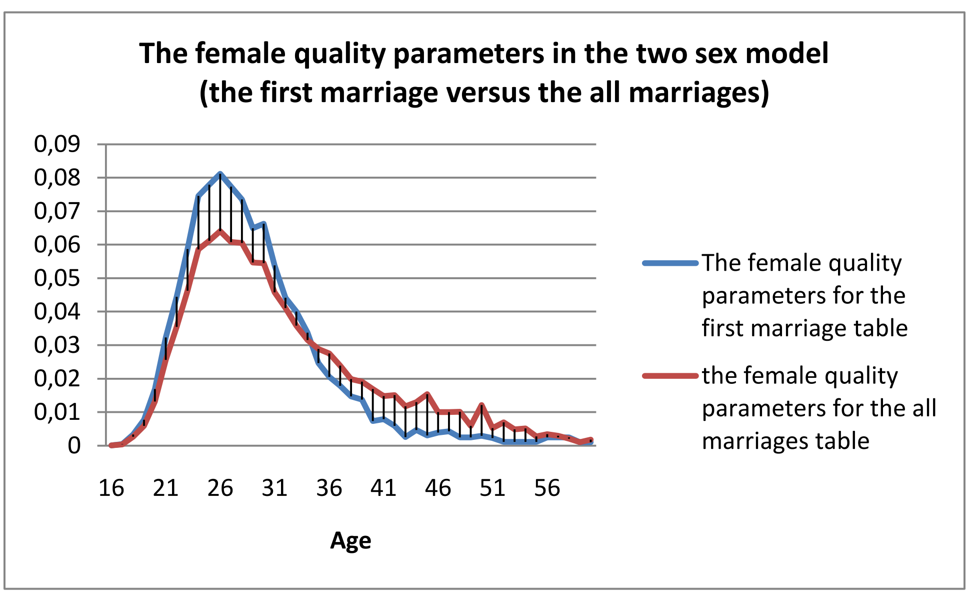


Figure 23



Now we turn to see the trends of gender quality parameters. In Figures 22 and 23 show the various gender quality parameters by age. For women, the curve for the all marriages is higher than that for first marriage table before age 36. I give the interpretation of those pictures from the point of marriage rate.

Here, I need to recall the formula for Ø

$$Ø_{ji} = \frac{M_{ji}}{V_{ji}} / \sum_{k=0}^{w-1} \frac{M_{jk}}{V_{jk}}, \quad j = 1, 2.$$

The overall marriage $\wedge_j$ rate is lower for first marriages than for all marriages, thus when computing Ø for all marriages one divides by a larger number. As we known, Ø represents the age-specific marriage rate scaled. For women at ages below 36, there are few remarriages; the marriage rate for first marriages is close to that for all marriages. Therefore, the curve of Ø for all marriages is lower than that for first marriages. For women at age over 36, rates for all marriages are higher than those for first marriages, which compensates the effect of a larger overall marriage intensity. Hence, after age 36, Ø for all marriages is higher. For men, the curves cross around age 40.

5 Conclusions

In this thesis I have estimated some of the parameters of a particular model for the two-sex marriage behavior developed by Juha Alho and Nico Keilman. In contrast to earlier work in this research field, this model takes the interaction between two sexes into account, and it displays real competition effects between marriage partners. It fulfills all the requirements as formulated in the so-called “McFarland Axioms”. Other models as outlined in Chapter 3 perform less adequately when the requirements for a realistic model are considered. Tables 3 and 4 present estimates for the marriage intensities in Norway based on annual marriage data for selected years in the period 1974-2002.

For the case of Norway, I compared first-marriage intensities computed for the two-sex marriage model with those based on a traditional one-sex marriage model (“first marriage table”)

The marriage intensity, being the one of the main indicators in our marriage models, has declined in Norway form 1974 to 1995, both for first marriages and all marriages, and both for males and females. Men and women married less often in the 1990s than in the 1970s and 1980, and when they did so, they did it at a higher age. The increasing popularity of consensual union is an important explanation for the trend in marriage behavior. Some cohabiting couples legalized their union after some years, others did not marry at all.

In Tables 3 and 4, female marriage intensities are consistently higher than male marriage intensities. Since one man marries one woman, this indicates that, there are fewer women available as marriage partners than men. Indeed, this was confirmed by empirical data on numbers of never-married and unmarried men and women.

Due to some higher order marriages people omitted, the two sex model results in relatively low intensities for first marriages. When the two-sex model is fitted to data on all marriages, higher intensities result.

Finally, my conclusion is that the two-sex marriage model developed by Juha Alho and Nico Keilman offers an accurate solution to the two-sex problem in the context of marriages. In this respect, the model displays genuine spill-over effects of the expected kind. It is more realistic than the traditional one-sex marriage model.

Table 3: The Values of Female and Male Marriage Intensities in the Two-Sex Marriage Model From 1974 to2002

	First Marriages	All Marriages
1974	FEMALE INTENSITY=2.1438	FEMALE INTENSITY=2.7578
	MALE INTENSITY= 1.7760	MALE INTENSITY= 2.4328
1980	FEMALE INTENSITY=1.3589	FEMALE INTENSITY=1.7031
	MALE INTENSITY= 1.1804	MALE INTENSITY= 1.6023
1985	FEMALE INTENSITY=1.0311	FEMALE INTENSITY=1.3749
	MALE INTENSITY= 0.9058	MALE INTENSITY= 1.3267
1990	FEMALE INTENSITY=0.9184	FEMALE INTENSITY=1.1951
	MALE INTENSITY=0.8043	MALE INTENSITY=1.1526
1995	FEMALE INTENSITY=0.8000	FEMALE INTENSITY=1.0928
	MALE INTENSITY=0.6970	MALE INTENSITY=1.0779
2000	FEMALE INTENSITY=0.9774	FEMALE INTENSITY=1.2781
	MALE INTENSITY=0.8301	MALE INTENSITY=1.2294
2001	FEMALE INTENSITY=0.8541	FEMALE INTENSITY=1.1300
	MALE INTENSITY=0.7224	MALE INTENSITY= 1.0815
2002	FEMALE INTENSITY=0.8881	FEMALE INTENSITY=1.1504
	MALE INTENSITY=0.7512	MALE INTENSITY=1.1006

Table 4: The Values of Female and Male Marriage Intensities of the First marriage in the One-Sex marriage Model from 1974 to 2002

Year	The value of Female Intensity	The Value of Male Intensity
1974	0.8832	0.8310
1980	0.7433	0.6930
1985	0.6435	0.5959
1990	0.6009	0.5527
1995	0.5507	0.5019
2000	0.6238	0.5640
2001	0.5744	0.5145
2002	0.5889	0.5282

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